Some useful equations from[stallings2007]

Some Basic Queuing Relationships

General		Single Server	Multiserver
$r = \lambda T_r$	Little's formula	$\rho = \lambda T_S$	$\rho = \dfrac{\lambda T_S}{N}$
$w = \lambda T_W$	Little's formula	$r = w + \rho$	$u = \lambda T_S = \rho N$
$T_r = T_W + T_S$			$r = w + N\rho$

Formulas for single-server queues

Assumptions:

1. Poisson arrival rate.
2. Dispatching discipline does not give preference to items based on service times.
3. Formulas for standard deviation assume first-in, first-out dispatching.
4. No items are discarded from the queue.

(a) General Service Times (M/G/1)

$$A = \frac{1}{2}\left[1 + \left(\frac{\sigma_{T_s}}{T_s}\right)^2\right]$$

$$r = \rho + \frac{\rho^2 A}{1-\rho}$$

$$w = \frac{\rho^2 A}{1-\rho}$$

$$T_r = T_s + \frac{\rho T_s A}{1-\rho}$$

$$T_w = \frac{\rho T_s A}{1-\rho}$$

(b) Exponential Service Times (M/M/1)

$$r = \frac{\rho}{1-\rho} \qquad w = \frac{\rho^2}{1-\rho}$$

$$T_r = \frac{T_s}{1-\rho} \qquad T_w = \frac{\rho T_s}{1-\rho}$$

$$\sigma_r = \frac{\sqrt{\rho}}{1-\rho} \qquad \sigma_{T_r} = \frac{T_s}{1-\rho}$$

$$\Pr[R = N] = (1-\rho)\rho^N$$

$$\Pr[R \le N] = \sum_{i=0}^{N}(1-\rho)\rho^i$$

$$\Pr[T_R \le T] = 1 - e^{-(1-\rho)\eta T_s}$$

$$m_{T_r}(y) = T_r \times \ln\left(\frac{100}{100-y}\right)$$

$$m_{T_w}(y) = \frac{T_w}{\rho} \times \ln\left(\frac{100\rho}{100-y}\right)$$

(c) Constant Service Times (M/D/1)

$$r = \frac{\rho^2}{2(1-\rho)} + \rho$$

$$w = \frac{\rho^2}{2(1-\rho)}$$

$$T_r = \frac{T_s(2-\rho)}{2(1-\rho)}$$

$$T_w = \frac{\rho T_s}{2(1-\rho)}$$

$$\sigma_r = \frac{1}{1-\rho}\sqrt{\rho - \frac{3\rho^2}{2} + \frac{5\rho^3}{6} - \frac{\rho^4}{12}}$$

$$\sigma_{T_r} = \frac{T_s}{1-\rho}\sqrt{\frac{\rho}{3} - \frac{\rho^2}{12}}$$

Formulas for multiserver queues (M/M/N)

Assumptions:

1. Poisson arrival rate.
2. Exponential service times.
3. All servers equally loaded.
4. All servers have same mean service time.
5. First-in, first-out dispatching.
6. No items are discarded from the queue.

$$K = \frac{\sum_{I=0}^{N-1} \frac{(N\rho)^I}{I!}}{\sum_{I=0}^{N} \frac{(N\rho)^I}{I!}} \qquad \text{Poisson ratio function}$$

$$\text{Erlang -C function} = \text{Probability that all servers are busy} = C = \frac{1-K}{1-\rho K}$$

$$r = C\frac{\rho}{1-\rho} + N\rho \qquad w = C\frac{\rho}{1-\rho}$$

$$T_r = \frac{C}{N}\frac{T_s}{1-\rho} + T_s \qquad T_w = \frac{C}{N}\frac{T_s}{1-\rho}$$

$$\sigma_{T_r} = \frac{T_s}{N(1-\rho)}\sqrt{C(2-C) + N^2(1-\rho)^2}$$

$$\sigma_w = \frac{1}{1-\rho}\sqrt{C\rho(1+\rho-C\rho)}$$

$$\Pr[T_w > t] = C\mathrm{e}^{-N(1-\rho)t/T_s}$$

$$m_{T_w}(r) = \frac{T_s}{N(1-\rho)}\ln\left(\frac{100C}{100-r}\right)$$

$$T_d = \frac{T_s}{N(1-\rho)}$$

Formulas for single-server queues with two priority categories

Assumptions:
1. Poisson arrival rate.
2. Priority 1 items are serviced before priority 2 items.
3. First-in, first-out dispatching for items of equal priority.
4. No item is interrupted while being served.
5. No items leave the queue (lost calls delayed).

(a) General Formulas

$$\lambda = \lambda_1 + \lambda_2$$

$$\rho_1 = \lambda_1 T_{s1}; \quad \rho_2 = \lambda_2 T_{s2}$$

$$\rho = \rho_1 + \rho_2$$

$$T_s = \frac{\lambda_1}{\lambda} T_{s1} + \frac{\lambda_2}{\lambda} T_{s2}$$

$$T_r = \frac{\lambda_1}{\lambda} T_{r1} + \frac{\lambda_2}{\lambda} T_{r2}$$

(b) Exponential Service Times

$$w_1 = \frac{\rho_1 \left(\rho_1 T_{s1} + \rho_2 T_{s2} \right)}{T_{s1} \left(1 - \rho_1 \right)}$$

$$w_2 = w_1 \frac{\lambda_2}{\lambda_1 (1 - \rho)}$$

$$T_{r1} = T_{s1} + \frac{\rho_1 T_{s1} + \rho_2 T_{s2}}{1 - \rho_1}$$

$$T_{r2} = T_{s2} + \frac{T_{q1} - T_{s1}}{1 - \rho}$$